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Third Semester B.E. Degree Examination, June / July 08
Signals and Systems

Time: 3 hrs.

Max. Marks:100

Note : 1. Answer any FIVE full questions.

2. Abbreviations : FT → Fourier Transform.

DTFT → Discrete - time Fourier transform.

- 1 a. Define the following : i) Time - invariance ii) Stability iii) Causality
iv) Invertibility. (04 Marks)
- b. A discrete - time system is characterized by the following input - output relationship :
 $Y(n) = x(n) + 3 u(n+1)$. Check the following properties : i) Stability ii) Causality
iii) Linearity iv) Time - invariance v) Memory. (08 Marks)
- c. The energy in a real - valued signal $x(n)$ is defined by, $E = \sum_{n=-\infty}^{\infty} x^2(n)$. Suppose that $x(n)$
has an even part defined by $\left(\frac{1}{2}\right)^{|n|}$ and the total energy of the signal $x(n)$ is 5J, find the
energy in the odd part of the signal. (08 Marks)
- 2 a. Convolute the two discrete - time sequences given below :
 $x_1(n) = \alpha^n u(n)$
 $x_2(n) = \alpha^{-n} u(-n)$. Take $|\alpha| < 1$. (08 Marks)
- b. Prove the following properties of convolution sum : i) Associativity ii) Distributivity
iii) Commutativity. (08 Marks)
- c. Find the frequency response of an LTI system whose impulse response is given below :
 $h(t) = -\delta(t+1) + \delta(t) - \delta(t-1)$. (04 Marks)
- 3 a. Consider an LTI system characterized by $h(n) = \frac{-n}{2} u(n)$.
i) If the system is excited by an input, $x(n) = 2\delta(n) + 4\delta(n-1) + 4\delta(n-2)$, find the
response of the system for $n = 0, 1, \dots, 6$. (10 Marks)
ii) Find the stability and frequency response of the system. (10 Marks)
- b. Evaluate the step response of the following LTI systems.
i) $h(t) = e^{-|t|}$ ii) $h(t) = u(t+1) - u(t-1)$. (10 Marks)
- 4 a. If $x(n) \xrightarrow{\text{DTFT}} X(e^{j\Omega})$ or $X(\Omega)$, show that $E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(\Omega)|^2 d\Omega$
(06 Marks)
- b. Find the DTFT of $x(n) = u(n)$. (07 Marks)
- c. Find the inverse DTFT of $X(\Omega) = X(e^{j\Omega}) = \frac{1}{[1 - a e^{-j\Omega}]^2}$. (07 Marks)

- a. If $x(t) \xrightarrow{FT} X(j\omega)$ or $X(\omega)$ and $y(t) \xrightarrow{FT} Y(j\omega)$ or $Y(\omega)$, show that

$$Z(t) = x(t) * y(t) \xrightarrow{FT} Z(\omega) = X(\omega) Y(\omega).$$

(07 Marks)

- b. Find the FT of $x(t) = \text{Sgn}(t)$. Also plot the magnitude and phase spectra. (07 Marks)

- c. Find the FT of the triangular pulse shown in fig.5(c). (06 Marks)

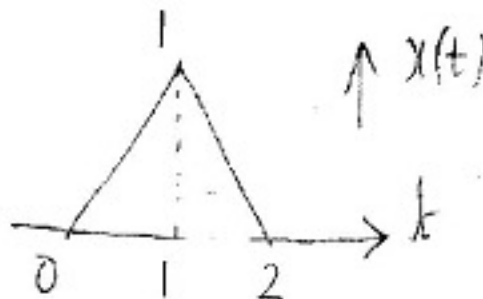


Fig.5(c)

- a. State and prove low pass sampling theorem. (10 Marks)

- b. Consider a continuous - time LTI system described by the following differential equation:

$$\frac{dy(t)}{dt} + 2y(t) = x(t). \text{ Find the following :}$$

i) Frequency response.

ii) Response of the system to an input $x(t) = e^{-t} u(t)$.

iii) Find the step response. (10 Marks)

- a. A causal discrete LTI system is described by $y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n)$. Find the frequency response and impulse response of the system. (07 Marks)

- b. State and prove final value theorem, using Z - transform. (07 Marks)

- c. Find the inverse Z - transform of $X(z) = e^{1/z}$. Take $x(n)$ to be a causal sequence. (06 Marks)

- a. Solve the difference equation, $y(n) = y(n-1) - y(n-2) + 2$, $n \geq 0$. Take $y(-2) = 1$ and $y(-1) = 2$. (10 Marks)

- b. A causal discrete - time LTI system is described by $y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n)$.

i) Find $H(z)$

ii) Find $h(n)$

iii) Find step response of the system.

iv) Find system stability

v) Find frequency response of the system. (10 Marks)
